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Fractal mass–size scaling of wetting soil aggregates

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Abstract

Structure is an important factor of soil functioning in ecosystems. Soil aggregate size distributions are commonly used to characterize soil structure. Relationships between density of dry soil aggregates and aggregate size present a different way to use aggregate-related information in soil structure characterization. Those relationships have been simulated assuming soils to be mass fractals. Aggregates in field soil are not air-dry. The relationships between mass and size differ between dry aggregates and wet aggregates because aggregates shrink as water content decreases. Our objective was to find out whether the mass fractal model can be applied to wet aggregates. Aggregates from the plow layer of Greyzem soil were brought to four different levels of water contents, and the kerosene method was used to measure volume of aggregates within diameter ranges of 3–5, 5–7, and 7–10 mm. It appeared that the wetter aggregates were less prone to loosening as the water increased. The mass fractal model was applicable to wet aggregates under the assumption of linear dependence of the fractal dimension D_m and the unit size aggregate mass a on the gravimetric water content w (g g^{-1}). Dependencies $D_m = 2.925 + 0.284w$ and $a = 0.808 - 0.123w$ resulted in $R^2 = 0.9999$ for the regression line of simulated versus measured aggregate mass. Fractal modeling of mass–size scaling in wet soil aggregates presents a set of aggregate-based parameters for soil structure that may reflect soil properties and can be explored as an index of soil ability to support functions of ecosystems.
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1. Introduction

Structure is an important factor of soil functioning in ecosystems. Soil structure has the major influence on the ability of soil to support plant growth, cycle carbon, and nutrients, receive, store, and transmit water, and to resist soil erosion and the dispersal of chemicals of anthropogenic origin. Particular attention must be paid to soil structure in managed ecosystems where human activities can cause both long-term and short-term changes that may have positive or detrimental impacts on the functions that soil fulfils (Kay and Angers, 2000).

Properties of soil aggregates are commonly used to characterize soil structure. Determining the aggregate size distribution presents one way of characterizing aggregate properties, and the distributions are obtained by sieving of air-dry soil through sieves with opening of different sizes and weighing the amount of soil at each sieve. Various parameters and functions to describe the shape of such distributions have been proposed and compared (Perfect et al., 1993). Relating the aggregate density to their size renders another type of information about aggregate properties. Wittmuss and Mazurak (1958) have probably been first to report that dry-aggregate density decreases as the aggregate size increases. Bartoli et al. (1991), Rieu and Sposito (1991), and Young and Crawford (1991) have found that the density–size relationships in air-dry aggre-

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gates follow predictions of models assuming aggregates to be mass fractals. The mass fractal dimension has proven to be useful in establishing the relationships between mass and size of dry aggregates. Eghball et al. (1993) showed that the value of mass fractal dimension is sensitive to tillage practices. Gimenez et al. (2002) demonstrated effects of tillage and erosion on the mass fractal dimension of air-dry soil aggregates.

Aggregate density and porosity changes during shrinking or swelling with water content change, and individual aggregate fractions may have different dependencies of aggregate size on water content (Voorhees et al., 1966; Chang and Warkentin, 1968). To our knowledge, the applicability of the mass fractal model to wet aggregates was never tested. The objective of this study was to determine whether fractal scaling is applicable to aggregates at various water contents.

2. Materials and methods

2.1. Soil data

Soil samples were taken from plow Ap, illuvial Ah, transitional EB and B horizons of Greyzem soil in the Vladimir region of Russia. Samples were air-dried and sieved to separate aggregates into size fractions with diameters 3–5, 5–7, and 7–10 mm. Particle size distribution, bulk density, particle density, and organic carbon content were measured in triplicate for each aggregate size fraction. Texture was measured with pipette method (Gee and Bauder, 1986) after dispersion with sodium pyrophosphate $Na_4P_2O_7$. Particles diameter ranges were <0.001, 0.001–0.005, 0.005–0.01, 0.01–0.05, 0.05–0.25, >0.25 mm. Particle density was measured with pycnometer method (Blake and Harge, 1986). Organic carbon content was measured with dry combustion method (Nelson and Sommers, 1996). Volume of air-dry aggregates was measured with the kerosene method (McIntyre and Stirk, 1954).

Relationship between aggregate bulk density and soil water content of soil aggregates was studied in plow (Ap) horizon with samples taken at depths 0–5, 5–10, 10–15, and 15–20 cm. Aggregates were dried at 105 °C during 24 h and placed on ceramic plate for capillary saturation. Water content of aggregates

was determined gravimetrically. Volume of individual aggregates was measured with kerosene method at air-dry water content, at two intermediate water contents between saturation and air-dry, and at saturation. Mass and volume of aggregates in each size range was measured in five replication at each water content.

2.2. Model of fractal mass–size scaling in wet aggregates

Let m and d be the mass of the aggregate solid phase (g), and the aggregate diameter (mm), respectively, within a size range. The aggregate diameter d changes as water content changes, value of m remains constant. We assume that the scaling relationship between masses and diameters of dry aggregates (Young and Crawford, 1991) can be expanded to include wet aggregates

$$m = a(w)d(w)^{D_m(w)} \tag{1}$$

Here and below w is the gravimetric water content (g g^{-1}), D_m the mass fractal dimension, and a is the mass of the solid phase of the unit-size aggregate having moisture content w (g). We further assume that dependencies of parameters a and D_m on water content w are linear

$$a(w) = a_1 + a_2 w \tag{2}$$

$$D(w) = D_{m1} + D_{m2} w \tag{3}$$

The equation

$$m = (a_1 + a_2 w)d(w)^{D_{m1} + D_{m2} w} \tag{4}$$

can be fitted to data on triplets (m, d, w) of measurements at different water content to find parameters a_1, a_2, D_{m1}, and D_{m2}. The SigmaPlot version 8 software (SPSS Inc.) was used for the fitting in this work.

Parameters a_1, a_2, D_{m1}, and D_{m2} may be used to simulate the aggregate shrinkage. Suppose that an aggregate has size d_a at water content w_a of air-dry soil. Since the mass m of the aggregate does not change, the size of the same aggregate at water content w will be

$$d(w) = \left\{ \frac{a(w_a)d_a^{D_m(w_a)}}{a(w)} \right\}^{1/D_m(w)} \tag{5}$$

It is convenient for our purposes to use the value of the specific porosity of aggregates η

$$\eta(w) = \frac{1}{\rho(w)} - \frac{1}{\rho_s} \qquad (6)$$

Here $\rho(w)$ is the aggregate density at water content w, and ρ_s is the solid particle density. Values of ρ are calculated directly from measured values

$$\rho(w) = \frac{m}{V(w)} \qquad (7)$$

where $V(w)$ is the aggregate volume at water content w. Using (1), Eq. (7) can be rewritten as

$$\rho(w) = \frac{a(w)}{c} d(w)^{D_m(w)-3} \qquad (8)$$

where c is the shape factor used to convert the cube of the diameter to the volume. Value $c = \pi/6$ was used in this work.

3. Results

Aggregates of different size within each soil horizon had similar texture. Aggregates in EB and B horizons contained more clay, and less silt and sand as compared with Ap and Ah horizons (Fig. 1). Soil in horizons Ap, Ah, and EB had the silt loam texture. The silty clay loam texture was found in horizon B. Texture of aggregates was similar to the soil texture in the Ap

horizon. Aggregates in Ah and EB horizons contained more clay as compared to bulk soil and were classified as silty clay loam. Aggregates from Ap and Ah horizons had higher organic carbon content (Table 1). Particle density in the aggregates generally increased with depth, but did not differ in Ah and EB. No significant difference in particle density was found between aggregate fractions (data not shown). Texture of bulk soil was markedly different from texture of aggregates in horizons EB and B.

Relationships between the aggregate air-dry mass and the aggregate size were similar within the pair of horizons Ap, Ah, and within the pair EB and B (Fig. 2). Slopes in Fig. 2 are steeper in transitional and B horizons where the largest values of D_m have been obtained (Table 1). Value of D_m equal three in B horizon indicates that all aggregates have equal density. The minimum value D_m of 2.81 was found in the Ah horizon.

Total of 415 triplets (m, d, w) of measurements at different water content made on samples from plow horizons were used to test the applicability of the relationships (2) and (3) to the aggregate mass scaling. Four parameters, a_1, a_2, D_{m1}, and D_{m2}, were found simultaneously by fitting Eq. (4) to data. The model fit the data in an excellent manner (Fig. 3) with $R^2 =$
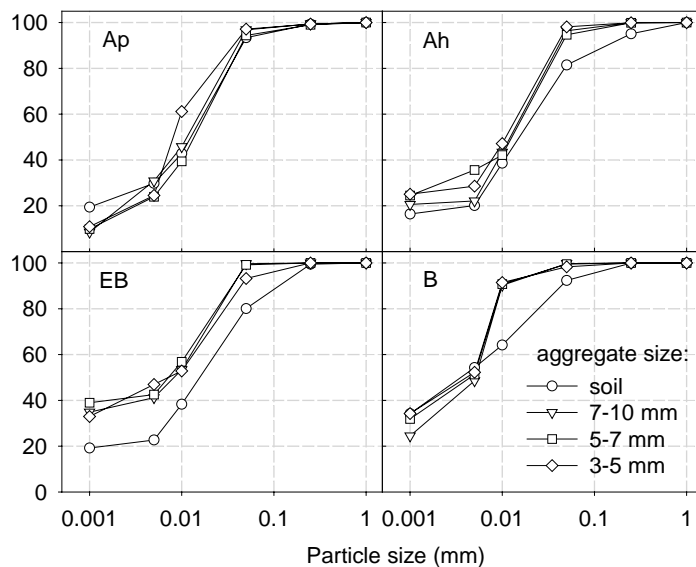

Fig. 1. Particle size distribution in soil horizons and in aggregates of different size.

Table 1
Some physical properties of Greyzem

Soil horizons	Ap	Ah	EB	B
Depth (cm)	0–26	26–52	52–68	68–100
Mass fractal dimension of air-dry aggregates	2.851 ± 0.014	2.806 ± 0.006	2.962 ± 0.004	3.000 ± 0.006
Aggregate size	Organic carbon content in aggregates (%)			
3–5 mm	2.625 ± 0.078	3.395 ± 0.049	0.635 ± 0.007	0.215 ± 0.021
5–7 mm	2.660 ± 0.099	3.420 ± 0.141	0.645 ± 0.007	0.225 ± 0.021
7–10 mm	2.600 ± 0.127	3.295 ± 0.007	0.625 ± 0.007	0.210 ± 0.000
Particle density in aggregates ($g\,cm^{-3}$)	2.617 ± 0.005	2.656 ± 0.004	2.662 ± 0.005	2.725 ± 0.012

0.9999, and the root mean square error of aggregate mass calculation was equal 0.022 g. Values of parameters in Eq. (4) were $a_1 = 0.808 \pm 0.0009$; $a_2 = -0.123 \pm 0.0048$; $D_{m1} = 2.925 \pm 0.0032$; $D_{m2} =$
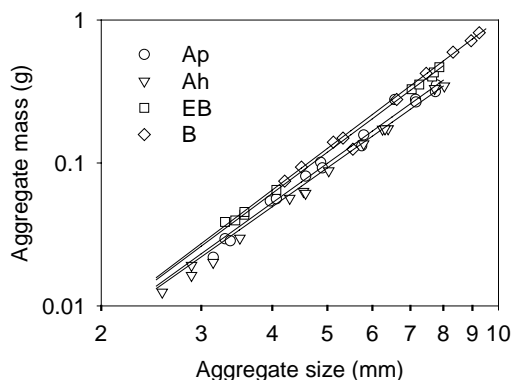

Fig. 2. Relationship between mass and size for aggregates from four horizons of the Greyzem soil.
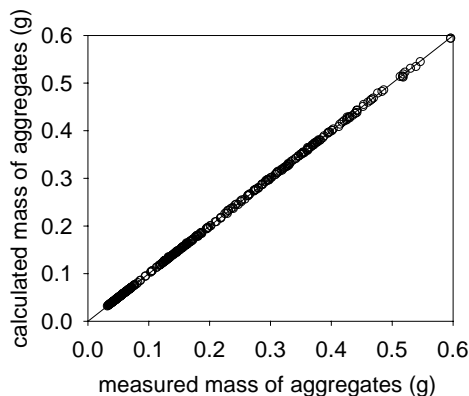

Fig. 3. Measured and calculated mass of aggregates at different water content.

0.284 ± 0.0175. The mass fractal dimension calculated from (3) for air-dry and water-saturated aggregates was equal to 2.932 ± 0.0001 and 3.002 ± 0.0001, respectively.

Parameters of the dependencies of D_m and a on the aggregate water content were used for calculation of shrinkage. Aggregate sizes of 3, 5, and 7 mm at air-dry water content were used for calculations of specific aggregate porosity using Eqs. (5)–(8). Comparison of measured and calculated shrinkage curves (Fig. 4) shows applicability of the fractal model to scale the aggregate mass in the range of water content from air-dry to saturation.
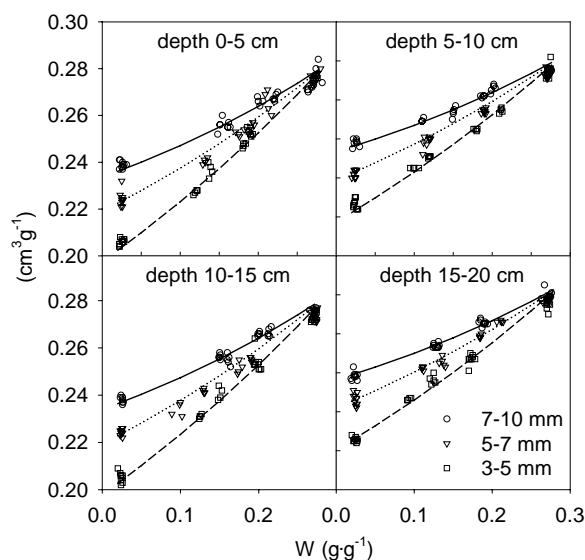

Fig. 4. Measured and calculated shrinkage curves of aggregates.

4. Discussion

Soils demonstrate many features that repeat themselves as the observation scale changes. The persistence of features across the hierarchy of scales is the probable reason for the applicability of fractal scaling that has been demonstrated for many soil properties (Crawford et al., 1999). Mass–size scaling of dry aggregates is one of manifestations of soil structural hierarchy. Values of mass fractal dimensions of dry aggregates in this study were within the range of values found by other authors. The mass fractal dimension D_m of a sandy loam soil (Carpow series) was 2.75 and 2.93–2.95 before and after cultivation, respectively (Young and Crawford, 1991). We computed values of D_m from data presented by Eghball et al. (1993) for Typic Argiudoll and found them in the range from 2.83 to 2.85. Gimenez et al. (2002) reported values of the mass fractal dimension for Typic Hapludults, Aquic Hapludults, and Ultic Hapludalfs soils. Values of D_m in their study varied greatly from 2.74 to 3.10 and were smaller in wooded soil as compared with cultivated. It has to be noted that the aggregate composition data can encompass only a relatively narrow range of scales.

Dry aggregate size distributions are known to show the influence of soil properties and water content during tillage on the failure zones along which soil breaks into aggregates (Kay and Angers, 2000). Fractal dimension of dry soil aggregates presents another aggregate-based parameter of soil structure that may reflect soil properties. In particular, increase in total organic carbon content usually results in an increase in size and stability of aggregates. This effect has been extensively reviewed (Kay and Angers, 2000). Soil organic carbon content may reflect the amount of polysaccharides that are expected to increase the cementation between mineral particles (Chenu and Guérif, 1991) and influence the arrangement of those particles between failure zones in soil. Data of this work show that organic carbon content is inversely related to the fractal dimension of dry aggregates (Table 1), thus, suggesting possible correlations between this fractal dimension and other basic soil properties.

We cannot provide the physics-based justification for the linear dependence of mass fractal dimension on water content, and the linearity may be just a first approximation of more complex dependence. Good results of application of the linear Eqs. (2) and (3) as shown in Fig. 3 present the empirical evidence that such approximation may be sufficient.

Aggregates ceased to be mass fractals at full saturation in this study. The maximum of scale-dependence is seen in air-dry aggregates where the fractal dimension reaches its minimum value. The scale-dependence interpreted with respect to soil pore space organization is important for soil organisms (Kampichler and Hauser, 1993), and there may be an optimum aggregate water content at which supply of water is still sufficient and satisfactory habitats for various groups of microorganisms still exist.

An increase in water content decreases the density of aggregates of unit size, and the value of the parameter a decreases. The density of unit size aggregates approaches the density of larger aggregates, which theoretically do not change at D_m equal to 3.00. Perhaps the swelling of clay contributes more to the aggregate porosity in small aggregates than in large ones.

We realize that some results of this work may be specific to the aggregate oven-drying prior to study volume–water content relationships. The oven-drying may cause irreversible changes in soil fabric (Tessier, 1990), and therefore, may influence the bulk density–water content scaling. The latter effect presents an interesting avenue to explore.

The water content in aggregates increased due to swelling of aggregates in this study. The opposite process of shrinking will not necessarily lead to the same density–water content dependence as during swelling, although hysteresis of shrinking–swelling has been documented for bulk soil but not for aggregates. Bronswijk and Evers-Vermeer (1990) observed aggregate shrinking for aggregates from 1.5 to 2 cm in diameter in seven soil profiles. He found that the aggregate volume decrease was equal to the water loss as aggregate dry from saturation to some intermediate water content between saturation and air-dry status. This meant that the mass fractal dimension remained equal to three and aggregates did not represent a mass fractal.

There exist recognized difficulties in relating soil structure to soil functions. Those difficulties are caused by the multiplicity of factors affecting soil structure and the multiplicity of effects the structure has on processes in soil. The search for informative parameters of soil structure continues, and is our hope that

research on aggregate scaling can generate useful complements to the existing parameterization of soil structure.

5. Conclusions

1. The mass fractal dimension of air-dry aggregates D_{m1} reflects hierarchical structure of genetic soil horizons. Minimum values of D_{m1} were obtained in the Ah horizon and maximum values were found in the B horizon. Mass fractal dimension increased as organic carbon content decreased.
2. The fractal scaling is valid for aggregates at water contents between air-dry and saturation. Value of the mass fractal dimension D_m reaches its maximum equal to three at saturated water content.
3. Both the mass fractal dimension D_m and the mass of the aggregate of the unit size a exhibited dependencies on gravimetric water content. Linear approximation of those dependencies rendered satisfactory results.
4. Dry aggregate size distributions are known to show the influence of soil properties and water content during tillage on the failure zones along which soil breaks into aggregates. Fractal modeling of mass–size scaling in wet soil aggregates presents another set of aggregate-based parameters for soil structure that may reflect soil properties and can be explored as an index of soil ability to support functions of ecosystems.

Acknowledgements

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